A Loudspeaker System Design Utilizing A Sixth-Order Butterworth Response Characteristic *

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A loudspeaker system of only 0.025 m³ internal volume capable of radiating an acoustic power in excess of 1/10 W at 32.7 Hz in pairs is described. Each complete system consists of the loudspeaker in its enclosure together with a complementary filter of the type described by Thiele as his alignment 15. Unusual features include a vent substitute appreciably larger than the main radiator.

INTRODUCTION: The republication of Thiele's elegant paper on vented-box loudspeaker systems [1] has caused the writer to take particular note of the possibilities inherent in properly executed systems of this type. This paper initially presents a review of the efficiency and cone excursion characteristics of direct-radiator loudspeaker systems and some of the ways in which these matters affect design. These considerations lead to the design of a specific system having a sixth-order Butterworth class I response. The resultant system is of a small-size format particularly suited to domestic use.

EFFICIENCY AND CONE EXCURSION CONSIDERATIONS

Thiele describes 28 selected methods of properly coordinating loudspeaker and box variables so as to permit the fabrication of vented-box systems to achieve specific response characteristics. By this time it is assumed that most individuals concerned with electroacoustic transducer design are acquainted with this information. Introduced in the article are relations describing direct-radiator conversion efficiency as a function of loudspeaker parameters and also cone excursion as a function of power output, frequency, loudspeaker size, and alignment specifics. These matters are of very special interest to a designer of loudspeakers because they are at the foundations of designing a system once maximum output level and cutoff frequency are established in the intended environment. In this manner an important chain relating output power (and hence sound pressure levels in a given listening environment) to required input power, cone excursion, and system type may be established.

Consider first the efficiency relationship for direct-radiator systems presented in Thiele [1, eq. (77)] as

$$\eta_{0b} = 8 \times 10^{-12} f_S^3 V_{AS}/Q_E \qquad (1)$$

where

- η_{0b} basic efficiency for radiation into 4π steradians (all space) at frequencies for which the wavelength is longer than the cone circumference (below about 700 Hz for an 8-inch loudspeaker)
- f_S free-air resonance of loudspeaker, in Hz
- V_{AS} air volume equivalent of acoustic stiffness of loudspeaker, in cubic inches
- Q_E Q of loudspeaker considering electromagnetic damping only.

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With certain assumptions it is possible to recast Eq. (1) into a form more directly related to the parameters of the loudspeaker system rather than to those of the loudspeaker alone. Small [2] and more recently the writer have used a form that may be written as

$$\eta_0 = 16 \times 10^{-12} f_3^{\,3} V_B K \qquad (2)$$

where

- η_0 efficiency for radiation into 2π steradians (half space)
- f_3 half-power (-3 dB) frequency of system relative to its constant output region
- V_B internal system volume, in cubic inches
- K efficiency factor dependent on system type.

The derivation of this equation is detailed in Appendix I. This relationship has been noted by several authors [3]–[5] and was recently commented upon in some detail by Small [6].

Perhaps the most striking matter indicated by Eq. (2) is the extreme penalty incurred in reaching for low frequencies with a given volume and K: every octave of response is paid for with 9 dB of conversion efficiency. Because extended low-frequency response coupled with moderate system size is usually of high desirability in domestic applications, it is of special interest to minimize the 9-dB per octave penalty. Eventually the payment of this penalty leads to excessive input power requirements and possible burnout of the loudspeaker.

An examination of the K factor indicates that unequalized sealed systems usually have values of 1 to 2, and unequalized vented systems (Thiele's alignments 1 through 9) have values of about 3 to 4. However, Thiele's sixth-order class I alignments (15 through 19) can have K values in the range of 9 to 18. These alignments require the use of auxiliary filters as a part of their concept with the filter chiefly functioning in the system's first octave. Peak boost values of 6 to 13 dB are required depending on the alignment used. It is this boost, which increases amplifier power output to compensate for falling loudspeaker system efficiency in the first octave, that accounts for high effective K values of these alignments.

Small [6] points out several useful ways of utilizing the K advantage of a fourth-order Butterworth vented system (Thiele's alignment 5) as compared to a second-order Butterworth sealed system:

(1) For the same V_B and f_3, efficiency may be increased by a bit over 4 dB.

(2) For the same efficiency and f_3, V_B may be reduced by a factor of almost 3.

(3) For the same efficiency and V_B, f_3 may be reduced by a factor of 0.73 or about one half octave.

If a similar comparison is made between a sixth-order Butterworth vented system (alignment 15) and a second-order sealed system, the efficiency increase is about 8 dB, the volume reduction factor is about 6, and the reduction in f_3 is a factor of about 0.55 or ⅞ octave. The additional advantages that are gained over alignment 5 in these areas are due to the increased K value of alignment 15 which is made possible by the use of the required auxiliary filter.

Thiele's alignment 19 offers the greatest differential

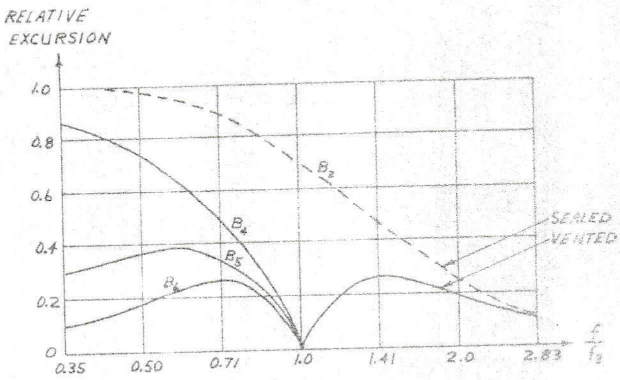

Fig. 1. Relative cone excursion versus frequency for Butterworth-aligned sealed and vented loudspeaker systems.

in a comparison of this type with an efficiency increase of 11 dB, a volume reduction factor of 13, and a possible f_3 reduction of a factor of 0.43 or 1¼ octave. The additional bonus in advancing from alignment 15 to 19 comes at the expense of an increase in maximum auxiliary circuit gain which must rise from 6 to over 13 dB. Also, alignment 19 (and several alignments preceding it) requires maximum auxiliary circuit boost to occur appreciably below the enclosure resonance frequency where cone excursions are rapidly climbing. These penalties are perhaps rather severe in view of the return.

The potential offered by increased K is rendered less valuable if a corresponding reduction in the cone excursion necessary to radiate a desired amount of power at a given low frequency is not also forthcoming. If, for example, an opportunity to reduce f_3 leads to cone excursions that are not attainable (or the required excursions lead to unacceptable distortion levels) a new dilemma is posed.

Fortunately relief is at hand as indicated in Thiele's equation for peak cone excursion [1, eq. (84)]:

$$x_{pk} = \frac{[1.31 \times 10^5 (W_o)^{1/2} / f_B^{\,2} S_D]}{[(f_b/f)^2 - (f_b/f)^4] \, |E(j\omega)|} \qquad (3)$$

where

- x_{pk} half the total peak to peak cone excursion, in inches
- W_o acoustic output, in watts
- f_B box tuning frequency, in Hz
- S_D piston area of speaker, in square inches
- $E(j\omega)$ response characteristic given by the desired system alignment.

A plot of normalized cone excursion versus normalized frequency for constant power input is shown in Fig. 1 for Butterworth vented-box alignments (solid curves) and also for a second-order Butterworth sealed system (dashed curve). Note that f_3 equals f_B for these responses.

The following observations may be made from the excursion information in the passband response region above f_3:

(1) The sealed system has its maximum excursion at f_3 while the vented system has its minimum placed at this point (the vented system maximum occurs at 1.45 f_3).

(2) The vented system maximum excursion in the

passband is only 0.35 that of the sealed system for the same acoustic output.

(3) Above about 3 f_3 the vent becomes inoperative causing the excursions for both system types to become the same for equal acoustic outputs.

The implications of the 0.35 ratio for maximum cone excursion in the loudspeaker system's passband is very important. Because of this the vented system can produce 9 dB more acoustic output for a given speaker size, f_3, and excursion limit; or the speaker cone diameter for the vented system can be 0.6 times that for the sealed system for the same f_3, acoustic output, and excursion limit; or further the f_3 for the vented system can be 0.6 times lower for a given speaker size, excursion limit, and acoustic output.

It is an interesting coincidence that the potential reduction of f_3 by a factor of 0.55 at constant system volume and efficiency described earlier (in comparing a Thiele alignment 15 to a second-order Butterworth system) is almost exactly matched by the ability to reduce f_3 by a factor of 0.6, based on cone excursion considerations alone. The implication clearly is that the use of a vented system with a sixth-order Butterworth characteristic to achieve a substantial reduction in f_3 not only does so without volume or efficiency penalties, but also maintains power output capability without any change in cone excursion requirements.

SOUND PRESSURE LEVEL REQUIREMENTS FOR HOME LISTENING ENVIRONMENTS

Just how much acoustic power is required for satisfactory listening levels in domestic environments? The first step in seeking an answer to this question is to note that peak values of sound pressure level (SPL) typical of many musical performances appear to be of the order of 100–120 dB. Olson [7] notes a value of about 100 dB "to render full artistic appeal." Stark [8], [9] suggests that levels approaching 115–120 dB may occur on high power transients and notes actual measurements on a pipe organ that are just shy of 105 dB on low-frequency peak pressures. Massa [10] suggests levels of the order of 110 dB (or 100 dyn/cm²) for realistic reproduction.

It may be verified through information available in Beranek [11, eq. (10.66)] that 110-dB SPL in the reverberant field of a room with a room constant of 200 ft² requires about 0.4 acoustic watt of loudspeaker output. Such a room could have a volume of the order of 3000 ft³ and might represent a "typical" living room environment for home listening.

Assume that occasionally all of a 110-dB peak level may be required near the region of peak cone excursion (admittedly a worst case type situation). Thiele [1, eq. (81)] derives an expression for peak cone excursion x_{pk} (in inches) for a sealed-box system considering radiation into 4π steradians or all space given as

$$x_{pk} = 1.31 \times 10^5 \sqrt{W_o}/f^2 S_D. \tag{4}$$

If it is further assumed that typically very low-frequency radiation from a pair of systems occurs into π to 2π steradians (¼ to ½ space, averaged here as ⅜ space radiation) and that 0.4-W total output is required with about 0.2-in peak (0.4-in total) excursion being available from each system before the onset of serious driver distortion, then Eq. (4) may be solved for df, where f is the frequency at which the 0.2-in peak excursion occurs and d is the piston diameter in inches. This derivation is detailed in Appendix II.

The solution for 110-dB (0.4-W) output is

$$df = 480. \tag{5}$$

The expression may similarly be solved for 105 dB (0.127 W required), yielding

$$df = 360. \tag{6}$$

Working from these expressions, Table I gives the calculated cutoff frequencies attainable for peak listening levels of 105 and 110 dB using pairs of sealed systems with 8-, 10-, or 12-in loudspeakers, having 0.4-in peak-to-peak cone excursion.

Oddly enough the 110-dB level results are reasonably typical of the resonance frequencies of many sealed systems. Additionally the 105-dB levels are fairly representative of several more recent sealed systems that have had their low-frequency limits pushed a bit lower than is perhaps usual. Conversion efficiencies of ¼ to 1 percent for ⅜ space radiation are often typical for small-form systems requiring 40–160-W peak input to radiate 0.4 W for the $df = 480$ systems. The $df = 360$ systems can be expected to be less than half as efficient and hence require a similar input power to achieve their lower maximum output for the same enclosure volumes.

It will be recalled from an earlier discussion that switching from a second-order Butterworth sealed system to a fourth-order or sixth-order (class I) Butterworth vented system would yield a potential reduction by a factor of 0.73 or 0.55 in f_3, respectively, based on keeping system volume and efficiency the same, and a reduction by a factor of 0.6 in f_3 based on keeping loudspeaker size and excursion the same. Taking the net possible reduction factor as 0.6 for the sixth-order Butterworth alignment reduces the df expressions to 290 for 110-dB SPL and to about 215 for 105-dB SPL. Keep in mind that peak passband excursion occurs at 1.45 f_3 for the vented system so that the f in the df expression becomes

Table 1. Frequencies at which 0.2-in (5mm) peak excursion occurs for paired sealed systems of various nominal sizes for 110- and 105-dB SPL. Also shown is the effect of a B_2 sealed to B_6 vented system conversion if the sealed system frequencies are construed to be low-frequency limits for a 0.2-in peak passband excursion.

Nominal Loudspeaker Diameter (inches)	Piston Diameter (inches)	Frequency at Which 0.2-in Peak Excursion Occurs for Sealed Systems		Sealed-System Frequencies Times 0.6 (as May Be Inferred from a B_2 Sealed to B_6 Vented System Conversion)	
		110-dB SPL	105-dB SPL	110-dB SPL	105-dB SPL
8	6½	74	55	45	33
10	8¼	58	44	35	26
12	10	48	36	29	22

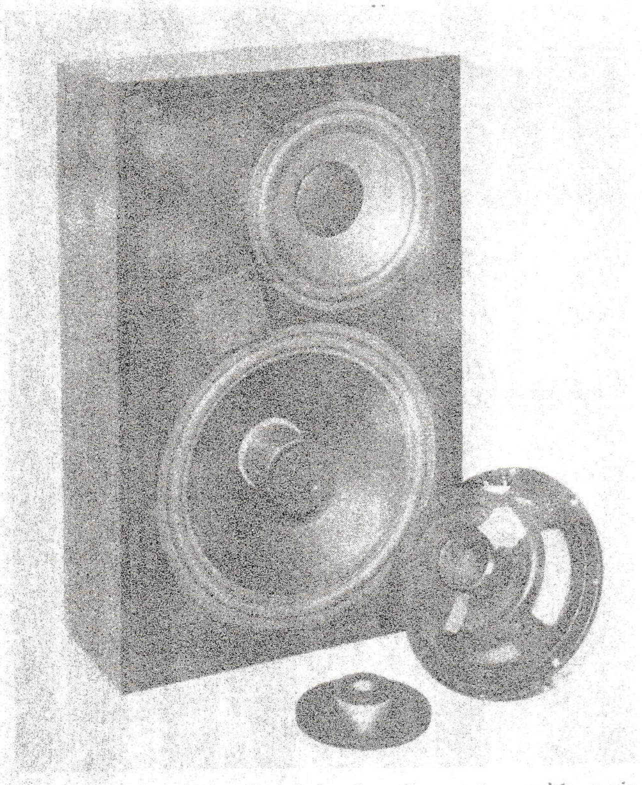

Fig. 2. Visual details of loudspeaker system with main loudspeaker also shown in partial assembly to illustrate interior details.

an equivalent low-frequency limit for the same peak passband excursion possessed by the sealed system. The last two columns in Table I list a new set of frequencies resulting from the 0.6 reduction factor being applied to the two preceding sealed-system cutoff frequency columns.

A SPECIFIC LOUDSPEAKER SYSTEM DESIGN

Examination of the preceding information led to an interest in creating a system capable of radiating reasonably high SPLs in domestic environments to C_1 (32.7 Hz) in a modest-size envelope (of the order of 1 ft^3) at a moderate but generally usable efficiency level. The approach involved using the smallest loudspeaker adequate for the purpose because this helps in maintaining constant power output to the highest possible frequency and fits readily into an enclosure of modest size. The small loudspeaker size lent itself to a two-way system design with a crossover frequency near 1500 Hz.

The 33-Hz potential of a 6½-in piston (8-in loudspeaker) in a sixth-order Butterworth vented system noted in the previous tabulation appears to be a most attractive solution for domestic environment SPLs of satisfactory value. Assuming about ¾-ft^3 internal volume, such a system provides ⅜ space conversion efficiencies in excess of 0.5%. The previous information suggests that Thiele's sixth-order Butterworth class I alignment 15 has the particularly interesting property of a high K value coupled with modest auxiliary circuit boost characteristics as well as the excursion advantages shared by all Butterworth-type vented-system alignments. It was considered desirable to build a system based on this alignment because of these potential performance advantages. In addition, a by-product of this alignment's required auxiliary filter is the 12-dB per octave electrical reduction of loudspeaker input below f_3. This reduces cone excursion in a frequency region where limited useful acoustic output is produced at the expense of distortion that can result from an overdriven loudspeaker. The filter also frees the amplifier from expending power where such expenditures can do little except add to distortion. It can also be shown that using alignment 15 instead of an unfiltered number 5 causes a smaller magnet mass to be required if enclosure size, f_3, and speaker size are the same, even though alignment 15 is approximately 4 dB more efficient.

A complete loudspeaker system designed on the above principles is shown in Fig. 2 together with some interior details of the 8-in loudspeaker which is shown in partial assembly in front of the system. One feature of the main loudspeaker is a ¾-in diameter hole through the magnet structure's center pole to relieve pressure buildup behind the dust cap. The loudspeaker is completely sealed using a solid dust cap and a nonporous polyurethane half-roll surround to avoid system leakage losses. Additionally it may be noted that the center pole is undercut with a conducting sleeve fitted over its entire length, except for the pole face. The sleeve, however, is included as a means of reducing second harmonic distortion which is caused by modulation of the speaker's permanent field by the voice coil signal [12].

A somewhat unusual feature of the system is the vent substitute in the form of a compliant weighted piston substantially larger than the main radiator. It might well be argued that there is little point in using the more complicated drone or vent substitute in place of a real vent unless port length and diameter are such as to cause excessive turbulence or viscosity losses. In the case of this system the writer never advanced to assessing these matters as the turbulence-induced wind noises (even with the application of several turbulence-reducing measures such as flanges and smooth surfaces) were judged to be excessive when using small vent volumes. Additionally, the vent volumes needed for even small vent areas were of the order of 25–40% of the actual box volume in order to tune the small enclosure to 32.7 Hz. Thiele's comments [1, section XI] relative to the undesirability of vent resistance was also strikingly evident at the high vent air velocities near f_B where even chicken wire vent coverings would noticeably reduce vent output and increase loudspeaker excursion appreciably, making the usage of grille cloth essentially impossible. A vent substitute was quickly subscribed to having properties of

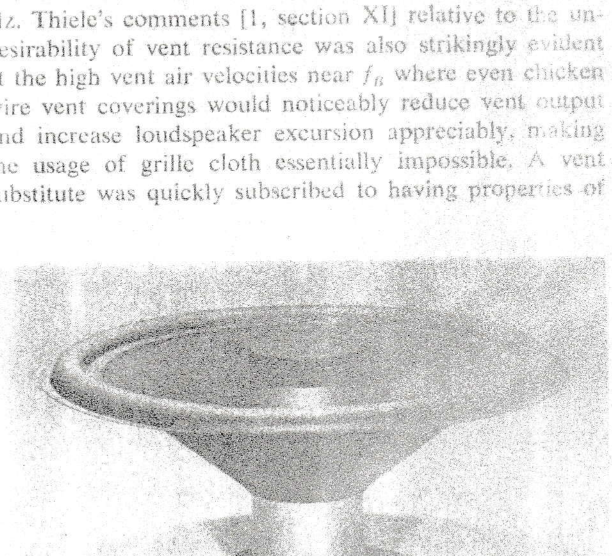

Fig. 3. Vent substitute removed from enclosure.

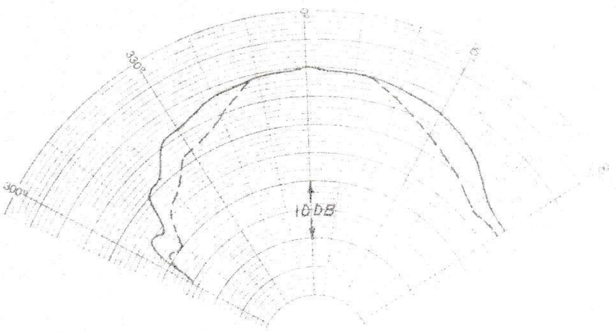

Fig. 4. Response at 15 kHz of tweeter with (solid) and without (dashed) directivity decreasing device.

low suspension stiffness, low suspension losses, and a moving mass equivalent to the weight of the air that would be contained in the real vent. With the preknowledge that at least 1/32 W could be radiated by the vent substitute near tuning, the sealed-box excursion equation noted earlier was used to determine that a 10-in piston would be required to move about 3/8 in peak to peak or more than 1/2 in if 1/16 W were to be exceeded. This piston size was adopted as being prudent.

Fig. 3 illustrates the vent substitute removed from the enclosure. The central steel tube serves as the weight and connector between the two suspension elements which are separated by over 5 in in order to ensure one-dimensional motion without appreciable rocking. The real vent that this tube replaces is 10 in in diameter and 20 ft long.

Another detail of the complete system that may be seen in Fig. 2 is a simple tweeter directivity decreaser. This consists of an absorber with a central hole and a carefully selected absorption characteristic which absorbs little in the lower tweeter range but reduces the tweeter's effective size at very high frequencies through increasing absorption of energy not passing through the central hole. Fig. 4 illustrates the decreased directivity in the vicinity of 15 kHz caused by this device. Necessary equalization to overcome absorber losses is incorporated into the active filter shown in Fig. 5. Not evident in Fig. 2 is a rear-mounted tweeter used primarily to touch up the total power output of the system in its upper octave to provide a nearly flat power output characteristic with low directionality. Fig. 6 shows both the power response and the axial anechoic response of the system as measured. A control incorporated into the equalizer allows the power characteristic to be rolled off to the extent of −3 and −6 dB at 10 kHz if the program material should require this. Although tweeter power handling is reasonable, provision is made on the rear of the system for installation of a mechanical tweeter disconnector that operates when input to either tweeter reaches about 6 W. Such a device may prove prudent when unusually high power amplifiers now becoming readily available are used, thereby avoiding difficulties inherent with aberrated spectra caused by tape rewinds at high speeds with the heads partially down, acoustic feedback, or just plain excessive input. The characteristics of the accessory disconnector are such as to not affect high power inputs of millisecond durations but to eventually disconnect at continuous average sine wave inputs of the order of 6 W if application is continued for 10's of milliseconds thus permitting short-term musical transients to be passed.

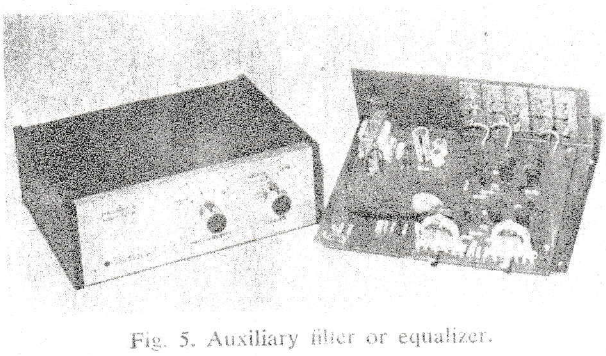

Fig. 5. Auxiliary filter or equalizer.

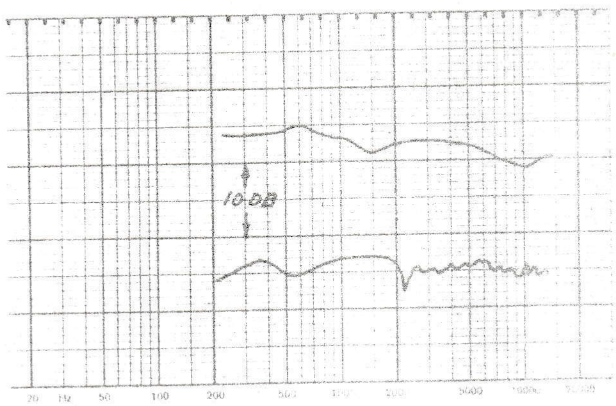

Fig. 6. The nature of power (top) and pressure outputs of loudspeaker system.

Shown in Fig. 7 is the power output available from the unit as governed by either the maximum allowed woofer excursion before serious distortion (about 0.35 in) or the maximum long-term power input to the woofer (about 30 W) or tweeters (5 W each). It should be emphasized that outputs are often shared between stereo pairs resulting in paired outputs perhaps being doubled over the displayed information in an average sense. Except in the region of maximum woofer excursion (near 45 Hz), the system should additionally be capable of handling short-term inputs of several times the long-term input capacities of the loudspeakers. In viewing Fig. 7 it should be kept in mind that an output of about 0.04 W is capable of producing 100-dB SPLs for the domestic room conditions presumed earlier.

ACKNOWLEDGMENT

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APPENDIX I

Thiele's efficiency equation, noted previously as Eq. (1), gives the nominal efficiency of a direct-radiator loudspeaker in terms of the three measured quantities f_S, V_{AS}, and Q_E. It is often more convenient or constructive to think of the efficiency in terms of loudspeaker system quantities instead. To do this it is first assumed that the major contributor to the loudspeaker Q_T is due to electromagnetic damping, making it possible to replace Q_E by Q_T so that Eq. (1) may be written as

$$\eta_{0b} = \frac{8 \times 10^{-12} f_S^3 V_{AS}}{Q_T}. \tag{7}$$

Thiele's C_{AS}/C_{AB} table entry may be replaced by $V_{AS}/V_B = a$ for enclosures not containing damping materials [11, eq. (5.38)]. If Thiele's table entry for $f_3/f_S = \beta$, the equation may be written as

$$\eta_{0b} = \frac{8 \times 10^{-12} f_3^3 V_B a}{\beta^3 Q_T}. \tag{8}$$

Calling $a/\beta^3 Q_T = K$ and rewriting for radiation into 2π steradians (½ space) would give the Eq. (2) form:

$$\eta_0 = 16 \times 10^{-12} f_3^3 V_B K. \tag{9}$$

The factor K may be evaluated for any of Thiele's alignments using the values of a, β, and Q_T given in [1, Table I].

In the case of sealed systems it is possible to write $f_3 = \gamma f_C$ (where f_C is the system resonance). K may then be written out as

$$K = \frac{a f_S^3}{\gamma^3 f_C^3} \frac{1}{Q_T}. \tag{10}$$

It may further be noted that if resonance frequency changes are presumed to be caused only by stiffness added by the enclosure, and if no damping materials are used, then from [11, eq. (8.13)],

$$\frac{f_C}{f_S} = \frac{Q_{TC}}{Q_T} = \sqrt{1 + \frac{C_{AS}}{C_{AB}}} = (1 + a)^{1/2} \tag{11}$$

where Q_{TC} is the system Q at f_C.

Substituting and rearranging then yields

$$K = \frac{a}{1 + a} \frac{1}{\gamma^3 Q_{TC}}. \tag{12}$$

If $a >> 1$ (a characteristic of many small sealed systems), then

$$K = \frac{1}{\gamma^3 Q_{TC}}. \tag{13}$$

For Butterworth response sealed systems ($\gamma = 1$ and $Q_{TC} = 1/\sqrt{2}$) this reduces to

$$K = \frac{1}{Q_{TC}} = \sqrt{2}. \tag{14}$$

APPENDIX II

It is possible to arrive at the $df = 480$ (110-dB SPL) and $df = 360$ (105-dB SPL) results by starting with [1, eq. (81)]

$$x_{pk} = \frac{1.31 \times 10^5 \sqrt{W_a}}{f^2 S_D}. \tag{15}$$

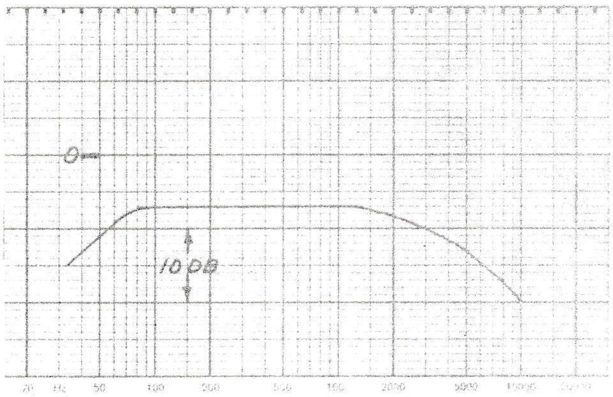

Fig. 7. Available power output of *one* loudspeaker system as limited by excursion or very long duration input power capacity. 0 dB is equivalent to 1 W output (approximately 114 dB in the "typical" environment described in the text). Note that two loudspeaker systems may yield about 3 dB more output.

This equation presumes radiation into 4π steradians. Since W_a is proportional to the radiation resistance R_R [11, eq. (7.5)] and R_R is essentially inversely proportional to the solid angle of radiation θ, the previous equation may be written as

$$x_{pk} = \frac{1.31 \times 10^5 \sqrt{W_a(\theta/4\pi)}}{f^2 S_D}. \tag{16}$$

By substituting $\pi d^2/4$ for the area of a circular piston, calling $\theta = 3/2\pi$ (⅜ space radiation) and setting x_{pk} at 0.2 in, the equation may be solved for df giving

$$df = 715 \sqrt{W_a}. \tag{17}$$

Assuming a room constant of 200 ft² and pressure domination by the reverberant sound field, it may be shown that [11, eq. (10.66)] SPLs of 110 dB and 105 dB represent outputs of 0.4 and 0.127 W, respectively.

If these total outputs are assumed to be randomly shared by two systems so that W_a is respectively 0.2 and 0.0635 W from an individual system, then df values of about 480 (110-dB SPL) and 360 (105-dB SPL) result.

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